Temperature Chaos in Two-Dimensional Ising Spin Glasses with Binary Couplings: a Further Case for Universality

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Abstract We study temperature chaos in a two-dimensional Ising spin glass with random quenched bimodal couplings, by an exact computation of the partition functions on large systems. We study two temperature correlators from the total free energy and from the domain wall free energy: in the second case we detect a chaotic behavior. We determine and discuss the chaos exponent and the fractal dimension of the domain walls.

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Introduction — A characteristic feature of spin glasses is the presence of chaos under small changes in the quenched couplings, in the temperature, or in the magnetic field [1, 2, 3, 4, 5, 6, 7, 8, 9]. With the expression *temperature chaos* we refer to the *fragility* of the equilibrium states of a disordered system under small temperature changes. Let us consider two typical equilibrium configurations of such a system under the same realization of the quenched disorder: the first configuration is in equilibrium at temperature T, while the second one is in equilibrium at temperature $T' = T + \Delta T$. One says that there is temperature chaos if for arbitrarily small (but non-zero) values of ΔT, the typical overlap of two configurations at T and T' goes to zero when the system size diverges. The spatial distance $\ell(T, \Delta T)$ over which such overlaps decay is called the *chaos length*, and, as we will discuss better in the following, it scales as $\ell \sim \Delta T^{-1/\zeta}$ when $\Delta T \to 0$.

The droplet picture [2, 10] of spin glasses predicts that

$$\zeta = \frac{d_s}{2} - \theta \, , \tag{1}$$

where d_s is the fractal dimension of the droplet interfaces and θ is the usual spin glass stiffness exponent. In two-dimensional ($2D$) spin glasses with *continuous* distributions of spin-spin couplings, one has $d_s \approx 1.27$ ([11]) and $\theta \approx -0.285$, leading to the prediction $\zeta \approx 0.92$; direct measurements of ζ [2, 12] are in good agreement with this prediction. The situation in the model with binary couplings ($J_{ij} = \pm 1$) is notably different: no study of temperature chaos has been performed, but if one works exactly at $T = 0$ using the estimates $d_s^{T=0} \approx 1.0$ ([13]) and $\theta^{T=0} = 0$ ([14, 15]), one would expect $\zeta^{T=0} \approx 0.5$ which is very different from the continuous distribution case, suggesting two universality classes. However, even though the $T = 0$ properties of spin glasses with continuous and discrete distributions are different, a new picture has recently emerged [16] in which one shows that *only one universality class* exists for $T > 0$ critical properties when we consider the limit $T \to 0$. In our work here we measure $\zeta^{T\to 0}$ in the $\pm J$ model and show that its value is compatible with the one of the model with Gaussian couplings, giving further credence to the generalized universality scenario [15, 16].

Our study is based on exact partition function computations for large but finite size systems, for many realizations of the quenched disordered couplings [17, 18]. Availability of the full density of states of a large system is a very powerful tool and it allows us to investigate the chaotic behavior of the free energy for the $\pm J$ model. We also extract entropies of domain walls and thus provide an improved estimate of the exponent d_s.

The outline of the paper is as follows. First we discuss the model, our observables and the methods we use. We discuss how to detect temperature chaos. We analyze what happens for the total free energy, finding there is no temperature chaos there. Then we move on to domain wall free energies. There we compute the two exponents, d_s from the entropy of $T = 0$ domain walls, and ζ from $T > 0$ scaling of two temperature correlators. We close by concluding that ζ has a value compatible with our generalized universality picture but not with the naive application of Eq. 1 (i.e. assuming $T = 0$ values for the domain-wall and chaos exponents).

Model, observables and methods — We analyze the $2D$ Edwards–Anderson spin glass with Ising spins ($\sigma_i = \pm 1$). Its Hamiltonian has the form

$$H_J(\sigma) \equiv - \sum_{<ij>} J_{ij} \, \sigma_i \, \sigma_j \, , \tag{2}$$

where the sum runs over all pairs of nearest neighbor sites on a square $2D$ lattice of linear size L with periodic boundary conditions. The couplings J_{ij} are independent quenched random variables taking the two values ± 1, each with probability 1/2.

To analyze the possible chaotic behavior of the system, we look at correlation functions of observables computed

at two different temperatures. For a generic observable $\mathcal{O}$ we use the correlation function

$$C_{\mathcal{O}}(L,T,\Delta T) \equiv \frac{\overline{(\mathcal{O}(T)-\overline{\mathcal{O}(T)})(\mathcal{O}(T')-\overline{\mathcal{O}(T')})}}{\sqrt{\overline{(\mathcal{O}(T)-\overline{\mathcal{O}(T)})^2}}\sqrt{\overline{(\mathcal{O}(T')-\overline{\mathcal{O}(T')})^2}}}\,, \tag{3}$$

where $T' \equiv T + \Delta T$, L is the linear size of the lattice and the bar stands for the quenched average over the distribution of the couplings J_{ij}. In the rest of this note we will focus on two main cases: in the first one $\mathcal{O}$ is the total free energy F of the system, while in the second case it is the domain wall free energy, F_{DW}.

We consider a $L \times L$ lattice with given quenched couplings J_{ij}, and we determine $F(T)$ and $F_{DW}(T)$ via the exact computation of the partition function Z [17, 18, 19]. We can then average over different samples to obtain (up to some statistical errors) the correlation functions $C_{\mathcal{O}}(L,T,\Delta T)$.

The total free energy — We start by discussing the computation of the free energy, where we identify $\mathcal{O}$ with the total free energy F of the system, and we compute $C_F(L,T,\Delta T)$. In a system where the total free energy has a chaotic behavior we would expect that in the large L limit C_F should drop very fast [20], as a function of L, even for infinitesimal values of ΔT. In the following we analyze C_F and we do not find a chaotic behavior. We use these results to show that even in the $T \to 0$ limit the total free energy does not behave chaotically in temperature.

We have averaged over a number of samples for different values of the system size $V \equiv L \times L$. Specifically, we have collected 99809 samples at $L = 10$, 31571 samples at $L = 20$, 3117 samples at $L = 30$, 818 samples at $L = 40$ and 336 samples at $L = 50$. Our error bars have been computed by using the jack–knife method (for details see [21]).

In our approach we obtain the full density of states, that allows us to compute expectation values at every temperature (or couple of temperatures) we are interested in. Because of that we select our temperature ranges (both for T and for ΔT) using the physical behavior of the system: we want to be for example as much as possible in a scaling regime, i.e. at low T, but not at very low T [19] (i.e. where, at given L, the system already shows an unphysical behavior dictated by the presence of a gap). Because of these facts we will present in our study data that span a range of temperatures going from $T = T_{min} \simeq 0.2$ to $T = T_{max} \simeq 0.4$: these values are chosen such that, for the considered linear sizes, we are in the scaling region where the relevant physical phenomena occur.

In figure 1 we plot $\ln C_F(L,T,\Delta T)$ as a function of ΔT, for $T = 0.25$; (that we know from [16, 19] to be in the scaling regime for $L \sim 50$); we use in the plot a ΔT step of 0.01 (reasonably small with respect to T). Somehow already this first, simple plot hints that the free energy of the system does not unveil a chaotic behavior.

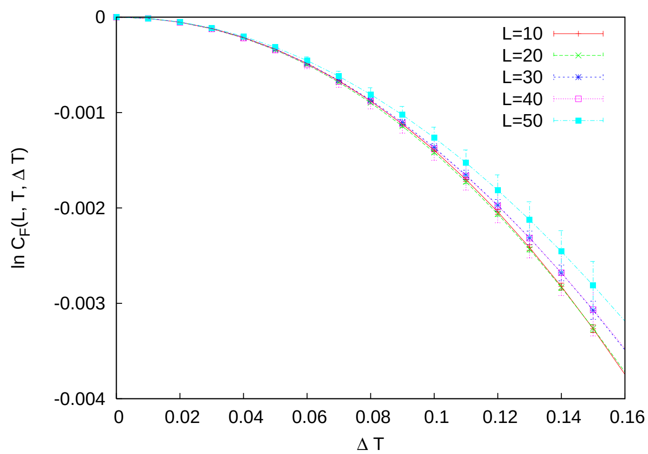

FIG. 1: $\log\{C_F(L,T,\Delta T)\}$ versus ΔT, for $T = 0.25$.

In fact one sees that the correlation function *rises* when L is increased. This trend is even more evident for larger values of ΔT. Such a behavior is the opposite of what we would expect in a chaotic scenario, where for large L values no correlation survives.

The issue of a possible presence of chaos in the free energy in the limit $T \to 0$ (i.e. when T approaches the $T = 0$ critical point) has to be dealt more carefully. For doing that we need a quantitative analysis of our data. We start by taking the infinite volume limit of our correlation functions. We present here the analysis of the correlation function with $T = 0.25$, but the other T values follow the same pattern. We have analyzed a number of temperatures in the scaling region that we are able to access (say for T going from 0.2 up to 0.4 [16, 19]). For each value of ΔT we fit $C_F(L,T = 0.25,\Delta T)$ to the functional form:

$$\begin{aligned} C_F(L,T=0.25,\Delta T) &= \widetilde{C}_F(T=0.25,\Delta T) + \\ &+ \frac{A(T=0.25,\Delta T)}{L}\,, \end{aligned} \tag{4}$$

determining in this way the large L limit $\widetilde{C}_F(T,\Delta T)$. As we noticed already, the correlation function increases with L: the fit to the form of Eq. 4 works very well, and C_F reaches a strictly positive asymptotic value in the thermodynamic limit.

From there, we are able to study $\widetilde{C}_F(T,\Delta T)$ as a function of ΔT (and, again, we do this for different values of T). A very good fit is obtained by assuming the functional dependence:

$$\widetilde{C}_F(T,\Delta T) = \exp\left(-a(T)\Delta T^2\right)\,. \tag{5}$$

In all cases this fit works very well; for example at $T = 0.25$ we estimate $a \simeq 0.114$. We find that in the limit $T \to 0$ the function $a(T)$ smoothly goes to a constant value (as opposed to its possible divergence, that would indicate chaos in the free energy): in the limit $T \to 0$ there is no chaos in the total free energy of the system.

Domain walls and their fractal dimension — Our second observable is based on domain walls. We introduce a domain wall into the system by applying to a given realization of the random quenched couplings first periodic boundary conditions (pbc) and then anti-periodic boundary conditions (apbc). For each sample, at any temperature, we define the domain-wall free energy as the difference of the free energies of these two systems; the same can be done for the energy and entropy. So for each realization of the couplings we compute the partition function twice: once with pbc and once with apbc (equivalently, we compute the partition function of two different systems with pbc, where in the second system we have inverted the sign of one line of couplings of the first).

Before discussing our results for this interesting correlation function we discuss a byproduct of this approach, that will be crucial in our final discussion of the physics of two-dimensional spin glasses. A nice feature of the exact partition function approach is that it gives us in particular the entropy of the domain wall even at zero temperature: this is a quantity that vanishes in the models with continuous J_{ij} coupling values, but that is non-zero in the model with binary quenched random couplings where the ground state degeneracy is high.

The zero temperature domain-wall entropy ΔS_{DW} has a disorder mean of zero (since inverting a number of couplings maps us to another sample in the disorder ensemble that has the same probability but has the opposite value of ΔS_{DW}). We thus focus on the mean square fluctuations; these have a characteristic size growing as

$$\overline{|\Delta S_{DW}|} \sim L^{\frac{d_s}{2}} \tag{6}$$

when $L \to \infty$. Previous work by Kardar and Saul [13] (also based on partition function computations, but with smaller statistics and for smaller L values than us) gave the estimation $d_s \approx 1.0$. We have used our data to obtain a more precise measurement of d_s. In Fig. 2 we show the scaling with L of the absolute value of ΔS_{DW} (to exhibit the very accurate power scaling). We also show our best fit which gives $d_s = 1.03 \pm 0.02$ (where we only quote the statistical error). This exponent d_s is usually interpreted as the fractal dimension of the domain wall created in the system; when $d_s > 1$, the domain wall is rough. However, in the $\pm J$ model there are many ground states and so it is not appropriate to think of d_s as associated with one interface; in fact it is not a priori necessary that $d_s \geq 1$. We note again that for Gaussian quenched random couplings one finds $d_s = 1.27$ [11, 12]; we thus see that having many domain walls in our degenerate system leads to smaller entropy fluctuations.

The chaos exponent ζ — Let us now discuss temperature chaos in the domain wall free energy F_{DW}, defined as the difference of the free energy in the system when using periodic boundary conditions versus anti–periodic boundary conditions. Given this quantity we use the definition (3) and we compute $C_{F_{DW}}$. Notice that the

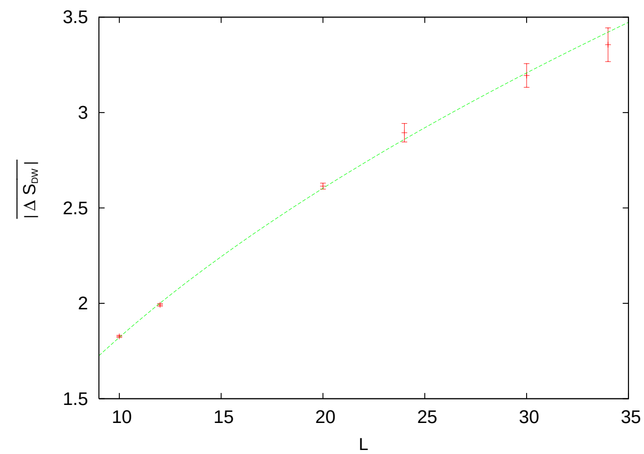

FIG. 2: $\overline{|\Delta S_{DW}|}$ versus L.

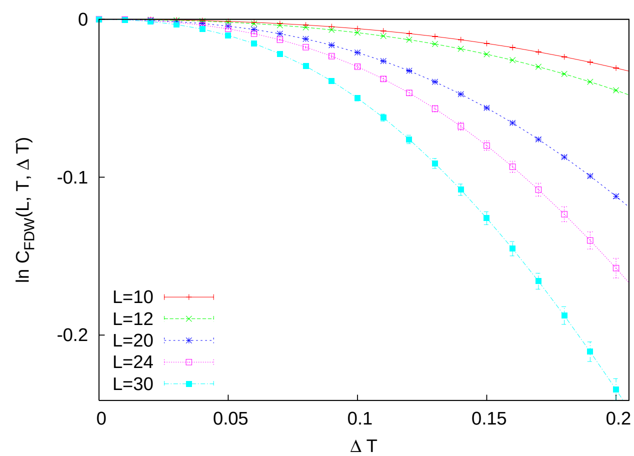

FIG. 3: $\log\{C_{F_{DW}}(L,T,\Delta T)\}$ versus ΔT, for $T = 0.25$.

relation (3) is simplified from the fact that $\overline{F_{DW}} = 0$.

Here we have used slightly smaller sizes than in the other case. For the linear sizes L = 10, 12, 20, 24, 30 we use different numbers of samples: 99808, 94351, 31570, 4098, 3116 respectively. In figure 3 we plot $\ln C_{F_{DW}}(L,T,\Delta T)$ as a function of ΔT for $T = 0.25$.

Fig. 3 is very different from Fig. 1: the linear correlation coefficient of the domain wall free energies at two different temperatures (with ΔT being fixed) decreases as L increases. As opposed to the case of the total free energy of the system, this trend is compatible with an emergent chaotic behavior. The same pattern arises when looking at the domain wall energy ($E_{DW} = E_{pbc} - E_{apbc}$) and at the domain wall entropy, ($S_{DW} = S_{pbc} - S_{apbc}$) and analyzing the related correlation functions $C_{E_{DW}}(L,T,\Delta T)$ and $C_{S_{DW}}(L,T,\Delta T)$.

The fact that the total free energy does not show any sign of chaos while the domain wall free energy hints for a possible chaotic behavior is, as we have already discussed, very natural. The small value of the exponent θ implies that in the domain wall free energy large cancellations play an important role [2] [3]. This suggests in turn

that the domain wall energy and the domain wall entropy (times T) exhibit a very similar temperature dependence and greatly contribute to this cancellation. This behavior has as a natural consequence the presence of temperature chaos: as soon as the parameters of the system get modified, even very slightly, cancellations will act in a different way and select completely different domain walls (the same arguments suggests the presence of chaos under a small change in the quenched disorder).

In the scaling theories, the domain wall free energy scales with the lattice size as L^{θ} (where θ is the stiffness exponent) and the domain wall entropy scales as $L^{d_s/2}$ (where d_s is the fractal surface of domain wall) [3]. Consider first the system at temperature T. Here the domain wall free energy scales with L as:

$$F_{DW}(T) \equiv E_{DW} - TS_{DW} \approx Y(T)L^{\theta} \;, \tag{7}$$

where $Y(T)$ is the *generalized stiffness coefficient*. Now if the system is at temperature $(T + \Delta T)$, the domain wall free energy scales as:

$$\begin{aligned} F_{DW}(T + \Delta T) &= E_{DW} - (T + \Delta T)S_{DW} \approx \\ &\approx Y(T)L^{\theta} - \Delta T L^{d_s/2} \;. \end{aligned} \tag{8}$$

Since the droplet theory predicts that $d_s/2 > \theta$, the quantity (8) for large L can change the sign between T and $T + \Delta T$. This implies that the equilibrium state changes when one reaches a length ℓ such that

$$\ell^{\theta} \sim \Delta T \; \ell^{d_s/2} \;, \tag{9}$$

that is

$$\ell \approx \left(\frac{1}{\Delta T}\right)^{\frac{1}{d_s/2-\theta}} \equiv \left(\frac{1}{\Delta T}\right)^{\frac{1}{\zeta}} \;. \tag{10}$$

It is important to note that these scaling laws are valid for small $T > 0$, and so both θ and d_s should be construed as being obtained at $T > 0$; this subtlety will be essential for interpreting our data.

When ΔT is small and all relevant length scales diverge one expects that [5]

$$C_{F_{DW}}(L, T, \Delta T) \approx 1 - \left(\frac{L}{\ell(T, \Delta T)}\right)^{2\zeta} \;, \tag{11}$$

for L smaller than $\ell(T, \Delta T)$, i.e. for small enough ΔT. In other words in this limit $1 - C_{F_{DW}}(L, T, \Delta T)$ scales as a power of L. In figure 4 we plot $(1 - C_{F_{DW}})$ vs. L on log-log scale for small values of ΔT ($\Delta T = 0.01$, 0.03, 0.05, 0.07), when $T = 0.25$ (similar results are obtained for other values of T). The scaling in L allows us to obtain ζ and $\ell(T, \Delta T)$ from a linear fit to $\ln(1 - C_{F_{DW}})$; we obtain ζ values between 0.92 and 0.99, depending on the values of T and ΔT.

We have also extracted from our fits the quantity $\ell(T, \Delta T)$. Fig. 5 shows that this length scale diverges as predicted in Eq. 10. The different best fit values for

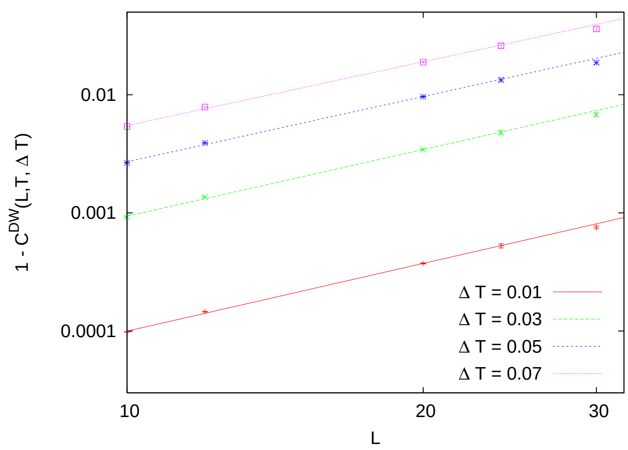

FIG. 4: $(1 - C_{F_{DW}})$ vs L in double logarithmic scale. Here $T = 0.25$.

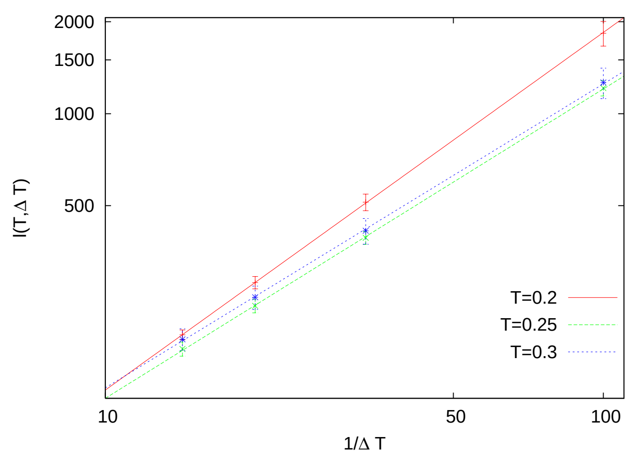

FIG. 5: $\ell(T, \Delta T)$ versus $\frac{1}{\Delta T}$ in double logarithmic scale.

ζ can be summarized by quoting $\zeta = 0.95 \pm 0.05$; we do not detect any systematic dependence on T.

In Fig. 6 we show an interesting data collapse. We plot $1 - C_{F_{DW}}$ as a function of $L/\ell(T, \Delta T)$, for $T = 0.25$ and $T = 0.3$ in double logarithmic scale. The fact that we get a single curve shows that the assumption that $C_{F_{DW}}(L, T, \Delta T)$ is only a function of $L/\ell(T, \Delta T)$ is reasonable. The fact that the curve is straight gives further support to Eq. 11.

Discussion and conclusions — This note spells a number of results. Let us start by summarizing them. First, when considering $2D$ Ising spin glasses with binary couplings, the free energy F does not show any temperature chaotic behavior, even in the $T \to 0$ limit. This is in contrast [20] with what happens in the directed polymer in a random medium, but in line with the prediction [9] of the Migdal-Kadanoff approximation. A simple interpretation is that the sample to sample non-chaotic fluctuations of F are $O(L)$, far larger than the chaotic $O(L^{\theta})$ fluctuations arising from droplets.

As opposed to the total free energy, the domain wall

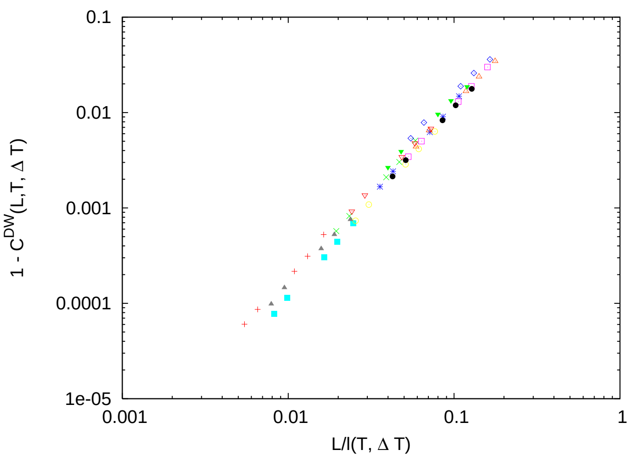

FIG. 6: $1 - C_{F_{DW}}$ versus $L/\ell(T, \Delta T)$, for $T = 0.2$, $T = 0.25$ and $T = 0.3$ in double logarithmic scale.

free energy does behave chaotically in T. The question of interest is whether the $\pm J$ model has chaotic behavior in the same class as the continuous distribution models. We first measured the fractal dimension of domain walls at $T = 0$ exactly and found $d_s^{T=0} \sim 1.0$. Then we measured directly the temperature chaos exponent ζ in the limit of small T and found $\zeta = 0.95 \pm 0.05$. (Note that these two measurements are conceptually very different.)

The point of view introduced by [15] and [16] is the following. If $T > 0$, all $2D$ Ising spin glasses, independently of the distributions of the quenched couplings, enjoy a strong, generalized universality [16]: a binary distribution of the couplings gives rise to the same critical behavior as the Gaussian distribution. Only exactly at $T = 0$ a very peculiar set of coupling distributions (including binary couplings) generates an anomalous behavior [15]: for cases where, by composing an arbitrary number of couplings, an energy gap survives (for example a distribution where $J = \pm 2, \pm 3$ is in this class, while a distribution where $J = \pm\sqrt{2}, \pm\sqrt{3}$ is not), then at $T = 0$ one gets $\theta = 0$. Our findings here for $T \to 0$ complete the scenario designed in [15, 16] and are completely consistent with it. If we use indeed the relation implied by the droplet theory $\zeta = d_s/2 - \theta$ and plug in both the value $\theta \sim -.285$ of the model with Gaussian quenched couplings and the value $d_s \sim 1.27$ obtained by [14] and by [11] we would expect $\zeta \sim 0.92$, in very good agreement with our estimate $\zeta^{T \to 0} = 0.95 \pm 0.05$. We believe that this is a very important point of support of the picture proposed and advocated in [15], [16]: there is one single critical theory that emerges, when $T \to 0$ in $2D$ spin glasses. The exponents of this theory are, at this point, well determined.

Our measurements also confirm that $T = 0$ is for the $J = \pm 1$ distribution, a singular point. In that model at $T = 0$, we found $d_s^{T=0} \sim 1.0$. This, together with the value $\theta = 0$, implies that, exactly at $T = 0$, if the relation $\zeta = d_s/2 - \theta$ were to be valid, we would have $\zeta^{T=0} = 1/2$. The difference of the physics at $T = 0$ and what one observes when $T \to 0$ (as defined from the order used for taking the $T \to 0$ and the $L \to \infty$ limits) is at this point self-evident.

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